Calculations policy 2016

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Examples of calculations methods for each year group and the progression between each method.

Our calculation policy

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This calculation policy has been created to meet the expectations of the new National Curriculum but most importantly the learning needs of our children here at Hallam Fields Junior School. The methods chosen match the National Curriculum but have also been specifically selected after consideration of our children's learning styles.

The policy is organised into year groups, considering the National Curriculum 2014 expectations. The new curriculum focuses on **skills** and **mastery** and is not about moving the children on to the next method as soon as they can do the one before. Working with more complex, varied and richer problems, rather than new methods, will support this 'mastering' of maths. Our whole school 'Non-negotiables' are included to maintain consistency in teaching, learning and assessment.

Written methods in maths have become increasingly important, but these will not replace the essential mental methods we have developed at Hallam Fields. These mental techniques remain the children's first method when approaching problems in maths.

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Problem solving

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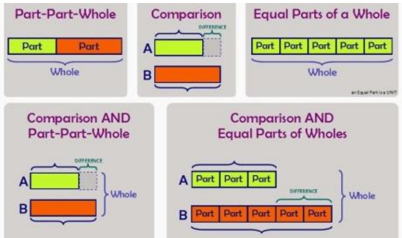
The new curriculum has a much greater focus on ***varied and frequent practice of the fundamentals*** of mathematics – including addition and subtraction facts and multiplication and division tables. However, the intention of the curriculum is that pupils will also be able to ***use and apply this knowledge in solving problems by reasoning mathematically.*** Reasoning mathematically and solving problems are requirements of the new curriculum.

When presented with a problem solving task children need to be familiar with RUCSAC (see below) and use this strategy to approach the problem step by step. We also teach the children the bar model approach which gives them powerful, but simple visual models they can draw upon and use to solve problems. All problem solving tasks need to be answered in context to ensure children have an understanding of the task set.

RUCSAC

Bar modelling



	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
Hallam Fields Junior School Growing together. Learning together. Achieving together ADDITION AND SUBTRACTION *Bonds & Facts* Hallam Fields Junior School Growing together. Learning together. Achieving together	Subtract multiple of 10 from 2 digit number. Represent and use number bonds and related subtraction facts within 20 Subtract practically 1 digit and 2 digit numbers to 20, including zero. Add practically 1 digit and 2 digit numbers to 20, including zero. Add & subtract two numbers under 10. Read, write and interpret mathematical statements involving addition, subtraction and equals signs.	Recognise and use inverse and use this to check calculations and solve missing number problems. To solve + and – problems using concrete objects and pictorial representations and using numbers, quantities and measures. Calculate the value of an unknown number in a number sentence. Add and subtract numbers mentally using TU+U, TU+T, TU+TU and U+U+U. Know and use related facts to 100. Know all number pairs to 100 using 'ten' numbers. Recall and use addition and subtraction facts to 20 fluently. Add & subtract numbers using concrete objects, pictorial representations	Create own function machine to show understanding of relationship. Demonstrate learning with concrete objects and own pictorial representations Estimate the answer to a calculation and use inverse operation to check. Solve varied + and – problems where answers may go above 100. Add and subtract numbers with up to 3 digits, using formal methods of column addition and subtraction. Add and subtract mentally, including: HTU+U, HTU+T and HTU+H. Calculate: HTU+TU and HTU–TU using number line Add & subtract using concrete objects, pictorial representations Know by heart addition & subtraction facts to 20.	Explain how inverse helps in specific real life situations. Explain why these are wrong… Create own two-step problems using key vocabulary. Demonstrate learning with concrete objects and own pictorial representations Solve + and – two-step problems in contexts, choosing operation, methods and explaining why. Add & subtract £ and pence using formal methods of column addition and subtraction. Add & subtract with 3-digit (progressing to 4-digit) using formal methods of column addition and subtraction. Use inverse operation to find missing numbers Add and subtract mentally pairs of 2 digit numbers (totals to add to over 100) Use any pairs of numbers that total 100 (and subtraction facts).	Create real life situations which involve using BOTH + and – Investigations involving balancing and proving ideas. Demonstrate learning with concrete objects and own pictorial representations Use knowledge of the order of operations to carry out calculations involving the four operations. Use rounding to make approximate answers to calculations and determine, in the context of a problem, levels of accuracy. Solve + and – multi-step problems in contexts, choosing operation, method and explaining why. Add & subtract whole numbers with up to 5 digits using formal written methods of column addition and subtraction accurately. Add & subtract to 2 dp. Add and subtract mentally with increasing numbers e.g 12, 462 – 2,300	Investigations involving balancing and proving ideas. Design a model to solve a practical situation. For every 2L of water I spend £… How much would I spend for …L? Investigations involving balancing and proving ideas. Add and subtract negative integers e.g. -7 – -5= Add and subtract positive and negative integers e.g. -7 + 5= Use estimation to check answers to problems and determine levels of accuracy. Add & subtract decimals to 3dp. Calculate: U.t±U.t Solve + and – multi-step problems in contexts, choosing operation, method and explaining why.

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Addition

Year 1

+ = signs and missing numbers

Children need to understand the concept of equality before using the '=' sign. Calculations should be written either side of the equality sign so that the sign is not just interpreted as 'the answer'.

2 = 1+ 1

2 + 3 = 4 + 1

Missing numbers need to be placed in all possible places.

3 + 4 = □   □ = 3 + 4

3 + □ = 7   7 = □ + 4

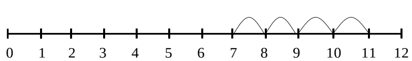
Counting and Combining sets of Objects

Combining two sets of objects (aggregation) which will progress onto adding on to a set (augmentation)

Understanding of counting on with a numbertrack.

Understanding of counting on with a numberline (supported by models and images).

7+ 4



Year 2

Missing number problems e.g 14 + 5 = 10 + □   32 + □ + □ = 100   35 = 1 + □ + 5

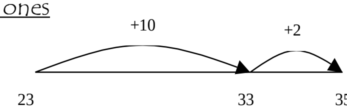
It is valuable to use a range of representations (also see Y1). Continue to use numberlines to develop understanding of:

Counting on in tens and ones

23 + 12 = 23 + 10 + 2

= 33 + 2

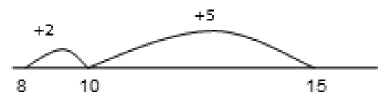
= 35



Partitioning and bridging through 10.

The steps in addition often bridge through a multiple of 10 e.g. Children should be able to partition the 7 to relate adding the 2 and then the 5.

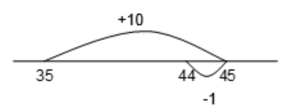
8 + 7 = 15



Adding 9 or 11 by adding 10 and adjusting by 1

e.g. Add 9 by adding 10 and adjusting by 1

35 + 9 = 44

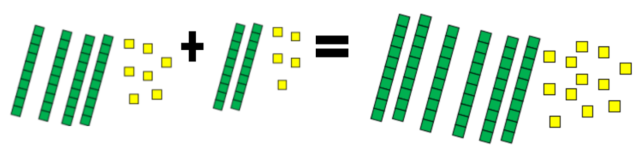


Towards a Written Method

Partitioning in different ways and recombine

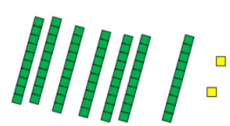
47+25

47   25   60 + 12



Leading to exchanging:

72



Expanded written method

40 + 7 + 20 + 5 =

40+20 + 7 + 5 =

60 + 12 = 72

40 + 7

+ 20 + 5

60 + 12 = 72

Year 3

Missing number problems using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones

Partition both numbers and recombine.

Count on by partitioning the second number only e.g.

247 + 125 = 247 + 100 + 20+ 5

= 347 + 20 + 5

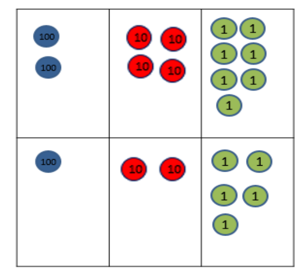
= 367 + 5

= 372

Children need to be secure adding multiples of 100 and 10 to any three-digit number including those that are not multiples of 10.

Towards a Written Method

Introduce expanded column addition modelled with dienes then move on to place value counters for a more abstract representation.



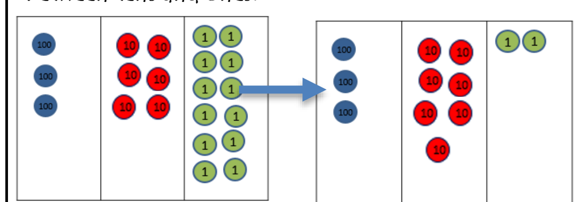
200 + 40 + 7

100 + 20 + 5

300 + 60 +12 = 372

```
  247
 +125
   12
   60
  300
  372
```

Leading to children understanding the exchange between tens and ones.



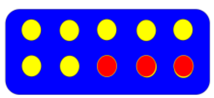
Some children may begin to use a formal columnar algorithm (up to 3 digits), initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

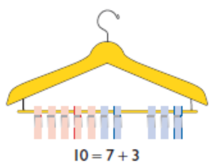
```
  247
 +125
  372
   10
```

Year 1

Mental Strategies (addition and subtraction)

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.
Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.
They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.





Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones and develop understanding of place value.
Children have opportunities to explore partitioning numbers in different ways.
e.g. 7 = 6 + 1, 7 = 5 + 2, 7 = 4 + 3 =
Children should begin to understand addition as combining groups and counting on.

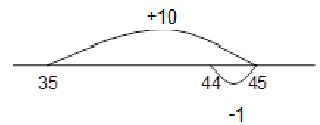
Vocabulary

Addition, add, forwards, put together, more than, total, altogether, distance between, difference between, equals = same as, most, pattern, odd, even, digit, counting on.

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting forwards in tens from any number should lead to adding multiples of 10.
Number lines should continue to be an important image to support mathematical thinking, for example to model how to add 9 by adding 10 and adjusting.



Children should practise addition to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 7 + 3 = 10 to find 17 + 3= 20, 70 + 30 = 100
They should use concrete objects such as bead strings and number lines to explore missing numbers 45+__= 50
As well as number lines, 100 squares could be used to explore patterns in calculations such as 74 +11, 77 + 9 encouraging children to think about 'What do you notice?' where partitioning or adjusting is used.
Children should learn to check their calculations, by using the inverse.
They should continue to see addition as both combining groups and counting on.
They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

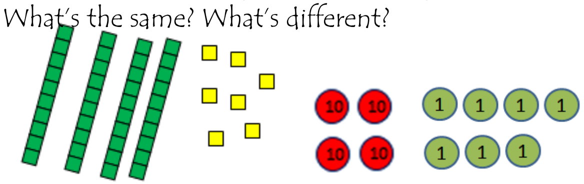
Vocabulary

+, add, addition, more, plus, make, sum, total, altogether, how many more to make…? how many more is… than…? how much more is…? =, equals, sign, is the same as, Tens, ones, partition
Near multiple of 10, tens boundary, More than, one more, two more… ten more… one hundred more

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. This will help to develop children's understanding of working mentally.
Children should continue to partition numbers in different ways.
They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g.
Add the nearest multiple of 10, then adjust such as 63 + 29 is the same as 63 + 30 – 1;
counting on by partitioning the second number only such as 72 + 31 = 72 + 30 + 1 = 102 + 1 = 103
Manipulatives can be used to support mental imagery and conceptual understanding. Children need to be shown how these images are related e.g.
What's the same? What's different?



Vocabulary

Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange
See also Y1 and Y2

Year 1

Generalisations

- True or false? Addition makes numbers bigger.
- True or false? You can add numbers in any order and still get the same answer.

(Links between addition and subtraction)
When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

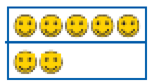
Some Key Questions

How many altogether? How many more to make…? I add …more. What is the total? How many more is… than…? How much more is…?
One more, two more, ten more…
What can you see here?
Is this true or false?
What is the same? What is different?

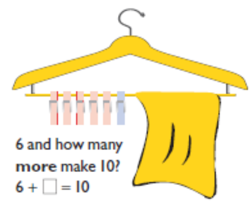
Year 2

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd + odd = even; odd + even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.



7 + ? = 10



Some Key Questions

How many altogether? How many more to make…?
How many more is… than…? How much more is…?
Is this true or false?
If I know that 17 + 2 = 19, what else do I know? (e.g. 2 + 17 = 19; 19 – 17 = 2; 19 – 2 = 17; 190 – 20 = 170 etc).
What do you notice? What patterns can you see?

Year 3

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
Odd + odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.
Addition and subtraction facts to 20
Solving problems where answers may go above 100.

Key Questions

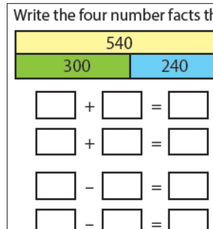
What do you notice? What patterns can you see?
When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line?

Mastery skills

What do you notice?
Is there a relationship between the calculations?

500 + 400 =	523 + 400 =	523 + 28 =
400 + 500 =	423 + 500 =	423 + 28 =
300 + 600 =	323 + 600 =	323 + 28 =
200 + 700 =	223 + 700 =	223 + 28 =
100 + 800 =	123 + 800 =	123 + 48 =

Write the four number facts that this bar model shows.



Using coins, find three ways to make £1.

Year 4	Year 5	Year 6

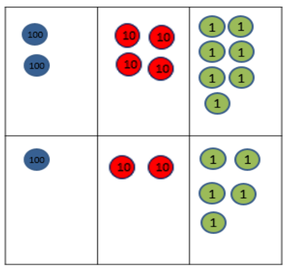
Year 4

Missing number/digit problems. Including using inverse to find missing numbers.

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

<u>Written methods (progressing to 4-digits)</u>

Expanded column addition modelled with place value counters, progressing to calculations with 4-digit numbers.

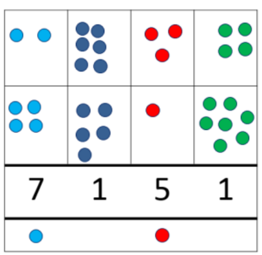


200 + 40 + 7
100 + 20 + 5
300 + 60 +12 = 372

```
  247
 +125
   12
   60
  300
  372
```

<u>Compact written method</u>

Extend to numbers with at least four digits.




```
  2634
 +4517
  7151
   1 1
```

Children should be able to make the choice of reverting to expanded methods if experiencing any difficulty.

Extend to up to two places of decimals (same number of decimals places) and adding several numbers (with different numbers of digits).

```
  72.8
+ 54.6
 127.4
 1 1
```

Year 5

Missing number/digit problems:

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving. Children should practise with increasingly large numbers to aid fluency e.g. 12462 + 2300 = 14762

<u>Written methods (progressing to more than 4-digits)</u>

As year 4, progressing when understanding of the expanded method is secure, children will move on to the formal columnar method for whole numbers and decimal numbers as an efficient written algorithm.

```
  172.83
+  54.68
  227.51
  1 11
```

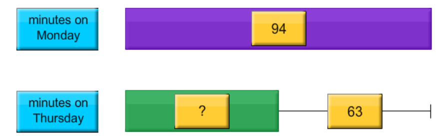
Place value counters can be used alongside the columnar method to develop understanding of addition with decimal numbers up to 2dp.

Year 6

Missing number/digit problems:

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

Jay practiced the trumpet 63 minutes longer on Monday than on Thursday. If he practiced for 94 minutes on Monday, how many minutes did he practice on Thursday?



<u>Written methods</u>

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with columnar method to be secured.

Add decimals to 3dp.

Continue calculating with decimals, including those with different numbers of decimal places

Calculate 36·2 + 19·8
- with a formal written column method
- with a mental method, explaining your reasoning

<u>Problem Solving</u>

Teachers should ensure that pupils have the opportunity to apply their knowledge in a variety of contexts and problems (exploring cross curricular links) to deepen their understanding. Multi step problems in contexts, choosing operation, method and explaining why.

Year 4

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: 28 + 75, 75 + 28 (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 5.6 + 3.7, 5.6 + 3 + 0.7 = 8.6 + 0.7
- Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating – 138 + 69, 138 + 70 - 1
- Partitioning: using 'near' doubles - 160 + 170 is double 150, then add 10, then add 20, or double 160 and add 10, or double 170 and subtract 10
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Vocabulary

add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Year 5

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.

Children should continue to partition numbers in different ways.

They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back - 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1
- Partitioning: using 'near' double: 2.5 + 2.6 is double 2.5 and add 0.1 or double 2.6 and subtract 0.1
- Partitioning: bridging through 60 to calculate a time interval: It is 11.45. How many hours and minutes is it to 15.20?
- Using known facts and place value to find related facts.

Vocabulary

tens of thousands boundary,
Also see previous years

Year 6

Mental Strategies

Consolidate previous years.

Children should experiment with order of operations, investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45

Put brackets in these number sentences so that they are true.

12 – 2 x 5 = 50

12 – 8 – 5 = 9

10 x 8 – 3 x 5 = 250

Vocabulary

See previous years

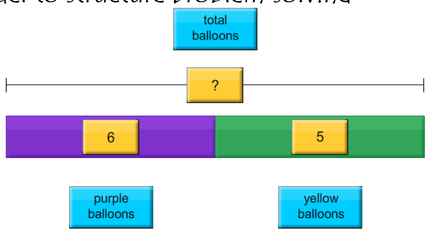
Year 4

Generalisations

Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Pairs of numbers that total 100

Solve up to 2 step problems in context – use bar model to structure problem solving



Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Mastery skills

2300	1240
3540	

☐ + ☐ = ☐

☐ + ☐ = ☐

☐ – ☐ = ☐

☐ – ☐ = ☐

Fill in the empty boxes to make the equations correct.

7 ☐ 1 + ☐ 3 ☐ = 999

7 ☐ 1 + ☐ 3 ☐ = 1000

Year 5

Generalisation

Use rounding to make approximate answers to calculations and determine levels of accuracy

Use knowledge of the order of operations to carry out calculations involving +, –, x and ÷

Solve addition multi step problems in context, choosing operation and explaining why.

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Mastery skills

Write four number facts that this bar diagram shows.

9·5	
3·8	5·7

☐ + ☐ = ☐

☐ + ☐ = ☐

☐ - ☐ = ☐

☐ - ☐ = ☐

Captain Conjecture says, 'When working with whole numbers, if you add two 2-digit numbers together the answer cannot be a 4-digit number.'

Do you agree?

Explain your reasoning.

Year 6

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS (Brackets, orders, division or multiplication, addition or subtraction) or could be encouraged to design their own ways of remembering.

Compare $31 + 9 \times 7$ and $(31 + 9) \times 7$

What's the same? What's different?

Use estimation to check answers to problems and determine levels of accuracy.

Add positive and negative integers (-7 + 5)

Add negative integers (-7 +-5)

Some Key Questions

What do you notice?

What's the same? What's different?

Can you convince me?

How do you know?

Mastery skills

Can you use five of the digits 1 to 9 to make this number sentence true?

☐ ☐ · ☐ + ☐ · ☐ = 31·7

Can you find other sets of five of the digits 1 to 9 that make the sentence true?

Can you use five of the digits 1 to 9 to make this number sentence true?

☐ ☐ · ☐ + ☐ · ☐ = 31·7

Can you find other sets of five of the digits 1 to 9 that make the sentence true?

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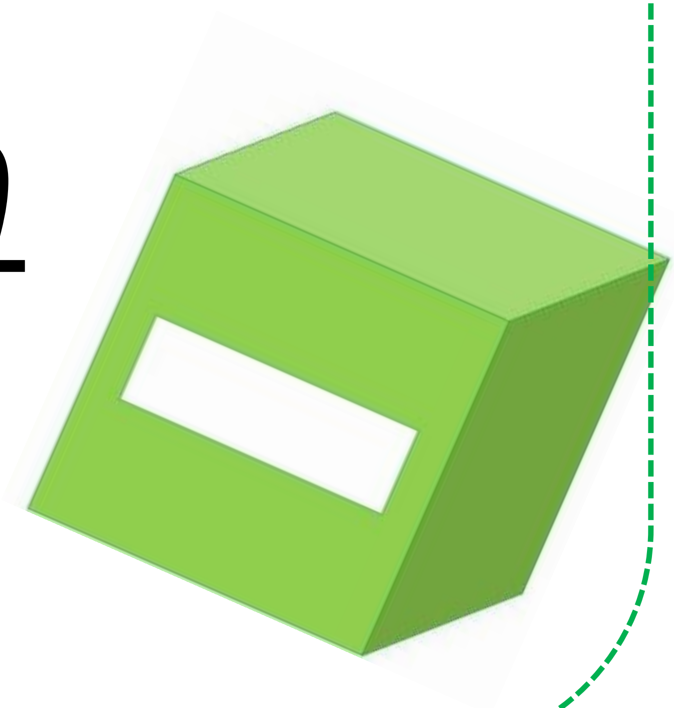
Subtraction

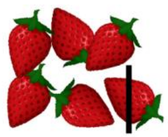


Year 1

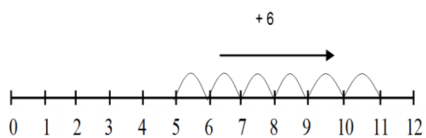
Missing number problems e.g. 7 = □ – 9; 20 – □ = 9; 15 – 9 = □; □ – □ = 11; 16 – 0 = □

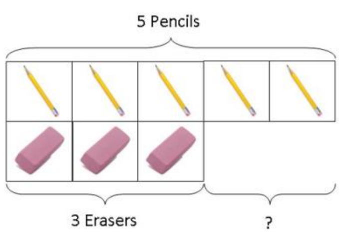
Use concrete objects and pictorial representations. If appropriate, progress from using number lines with every number shown to number lines with significant numbers shown.

Understand subtraction as take-away:



Understand subtraction as finding the difference:





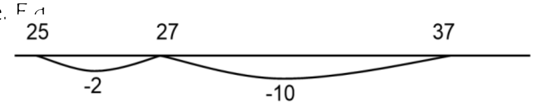
The above model would be introduced with concrete objects which children can move (including cards with pictures) before progressing to pictorial representation.

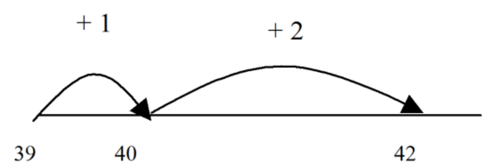
The use of other images is also valuable for modelling subtraction e.g. Numicon, bundles of straws, Dienes apparatus, multi-link cubes, bead strings

Year 2

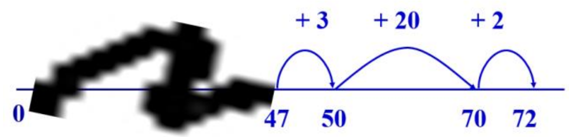
Missing number problems e.g. 52 – 8 = □; □ – 20 = 25; 22 = □ – 21; 6 + □ + 3 = 11

It is valuable to use a range of representations (also see Y1). Continue to use number lines to model take-away and difference. E.g.





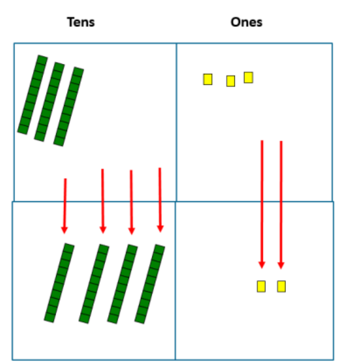
The link between the two may be supported by an image like this, with 47 being taken away from 72, leaving the difference, which is 25.



The bar model should continue to be used, as well as images in the context of **measures**.

<u>Towards written methods</u>

Recording addition and subtraction in expanded columns can support understanding of the quantity aspect of place value and prepare for efficient written methods with larger numbers. The numbers may be represented with Dienes apparatus. E.g. 75 – 42



	70	5
-	40	2
	30	3

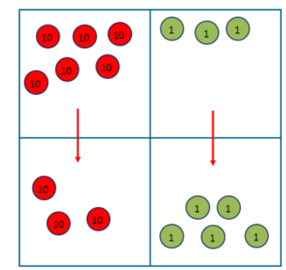
Year 3

Missing number problems e.g. □ = 43 – 27; 145 – □ = 138; 274 – 30 = □; 245 – □ = 195; 532 – 200 = □; 364 – 153 = □

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving (see Y1 and Y2). Children should make choices about whether to use complementary addition or counting back, depending on the numbers involved.

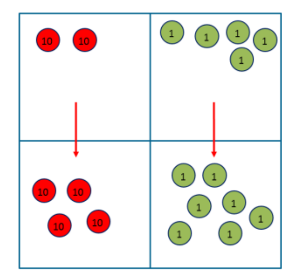
<u>Written methods (progressing to 3-digits)</u>

Introduce expanded column subtraction with no decomposition, modelled with dienes and moving on to place value counters for a more abstract representation.



	90	8
-	30	5
	60	3

For some children this will lead to exchanging, modelled using place value counters (or Dienes).



	~~70~~ 60	[1]2
-	40	7
	20	5

A number line and expanded column method may be compared next to each other.

Some children may begin to use a formal columnar algorithm (up to 3 digits), initially introduced alongside the expanded method. The formal method should be seen as a more streamlined version of the expanded method, not a new method.

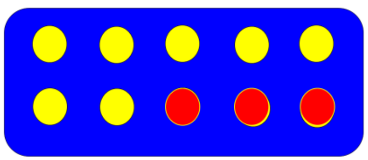
Year 1

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.

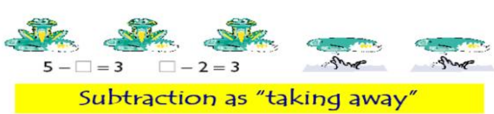
Children should memorise and reason with number bonds for numbers to 20, experiencing the = sign in different positions.

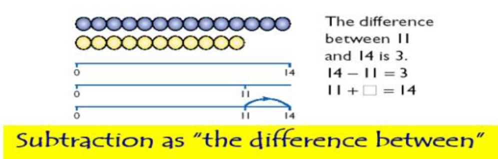
They should see addition and subtraction as related operations. E.g. 7 + 3 = 10 is related to 10 – 3 = 7, understanding of which could be supported by an image like this.



Use bundles of straws and Dienes to model partitioning teen numbers into tens and ones.

Children should begin to understand subtraction as both taking away and finding the difference between, and should find small differences by counting on.



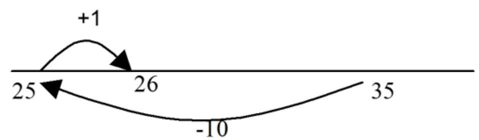


Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10. Counting back in tens from any number should lead to subtracting multiples of 10.

Number lines should continue to be an important image to support thinking, for example to model how to subtract 9 by adjusting.



Children should practise subtraction to 20 to become increasingly fluent. They should use the facts they know to derive others, e.g using 10 - 7 = 3 and 7 = 10 - 3 to calculate 100 - 70 = 30 and 70 = 100 - 30.

91	92	93	94	95	96	97	98	99	100
81	82	83	84	85	86	87	88	89	90
71	72	73	74	75	76	77	78	79	80
61	62	63	64	65	66	67	68	69	70
51	52	53	54	55	56	57	58	59	60
41	42	43	44	45	46	47	48	49	50
31	32	33	34	35	36	37	38	39	40
21	22	23	24	25	26	27	28	29	30
11	12	13	14	15	16	17	18	19	20
1	2	3	4	5	6	7	8	9	10

As well as number lines, 100 squares could be used to model calculations such as 74 – 11, 77 – 9 or 36 – 14, where partitioning or adjusting are used. On the example above, 1 is in the bottom left corner so that 'up' equates to 'add'.

Children should learn to check their calculations, including by adding to check.

They should continue to see subtraction as both take away and finding the difference, and should find a small difference by counting up.

They should use Dienes to model partitioning into tens and ones and learn to partition numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.

Year 3

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.

The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged.

Children should continue to partition numbers in different ways.

They should be encouraged to choose the mental strategies which are most efficient for the numbers involved, e.g. counting up (difference, or complementary addition) for 201 – 198; counting back (taking away / partition into tens and ones) for 201 – 12.

Calculators can usefully be introduced to encourage fluency by using them for games such as 'Zap' [e.g. Enter the number 567. Can you 'zap' the 6 digit and make the display say 507 by subtracting 1 number?]

The strategy of adjusting can be taken further, e.g. subtract 100 and add one back on to subtract 99. Subtract other near multiples of 10 using this strategy.

Year 1

Vocabulary

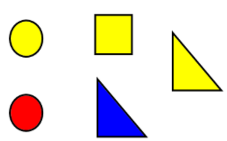
Subtraction, subtract, take away, distance between, difference between, more than, minus, less than, equals = same as, most, least, pattern, odd, even, digit,

Generalisations

- True or false? Subtraction makes numbers smaller
- When introduced to the equals sign, children should see it as signifying equality. They should become used to seeing it in different positions.

Children could see the image below and consider, "What can you see here?" e.g.

3 yellow, 1 red, 1 blue. 3 + 1 + 1 = 5
2 circles, 2 triangles, 1 square. 2 + 2 + 1 = 5
I see 2 shapes with curved lines and 3 with straight lines. 5 = 2 + 3
5 = 3 + 1 + 1 = 2 + 2 + 1 = 2 + 3



Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many have gone? One less, two less, ten less… How many fewer is… than…? How much less is…?
What can you see here?
Is this true or false?

Year 2

Vocabulary

Subtraction, subtract, take away, difference, difference between, minus
Tens, ones, partition
Near multiple of 10, tens boundary
Less than, one less, two less… ten less… one hundred less
More, one more, two more... ten more... one hundred more

Generalisation

- Noticing what happens when you count in tens (the digits in the ones column stay the same)
- Odd – odd = even; odd – even = odd; etc
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems. This understanding could be supported by images such as this.

15 + 5 = 20

Some Key Questions

How many more to make…? How many more is… than…? How much more is…? How many are left/left over? How many fewer is… than…? How much less is…?
Is this true or false?
If I know that 7 + 2 = 9, what else do I know? (e.g. 2 + 7 = 9; 9 – 7 = 2; 9 – 2 = 7; 90 – 20 = 70 etc).
What do you notice? What patterns can you see?

Year 3

Vocabulary

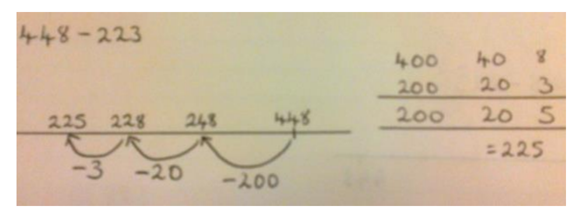
Hundreds, tens, ones, estimate, partition, recombine, difference, decrease, near multiple of 10 and 100, inverse, rounding, column subtraction, exchange
See also Y1 and Y2

Generalisations

Noticing what happens to the digits when you count in tens and hundreds.
Odd – odd = even etc (see Year 2)
Inverses and related facts – develop fluency in finding related addition and subtraction facts.
Develop the knowledge that the inverse relationship can be used as a checking method.
Addition and subtraction facts to 20

Key Questions

What do you notice? What patterns can you see?
When comparing two methods alongside each other: What's the same? What's different? Look at this number in the formal method; can you see where it is in the expanded method / on the number line



Mastery skills

Flo and Jim are answering a problem:
Danny has read 62 pages of the class book, Jack has read 43. How many more pages has Danny read than Jack?
Flo does the calculation 62 + 43. Jim does the calculation 62–43.
Who is correct?

Explain how you know.

Pupils might demonstrate using a bar model to explain their reasoning.

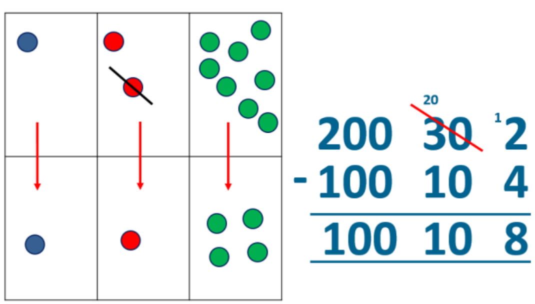
Year 4

Missing number/digit problems: 456 + □ = 710; 1□7 + 6□ = 200; 60 + 99 + □ = 340; 200 – 90 – 80 = □; 225 - □ = 150; □ – 25 = 67; 3450 – 1000 = □; □ - 2000 = 900

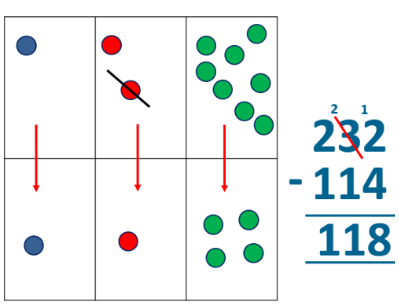
<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

<u>Written methods (progressing to 4-digits)</u>

Expanded column subtraction with decomposition, modelled with place value counters, progressing to calculations with 4-digit numbers.



If understanding of the expanded method is secure, children will move on to the formal method of decomposition, which again can be initially modelled with place value counters.



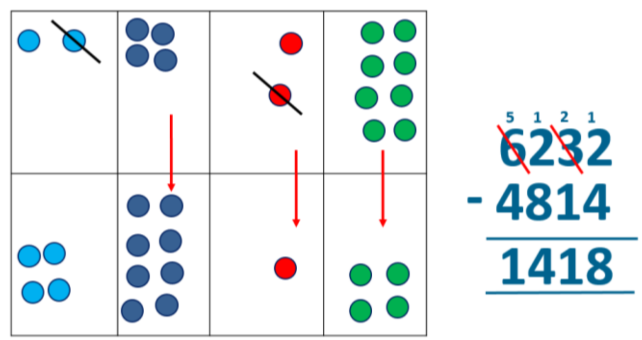
Year 5

Missing number/digit problems: 6.45 = 6 + 0.4 + □; 119 – □ = 86; 1 000 000 – □ = 999 000; 600 000 + □ + 1000 = 671 000; 12 462 – 2 300 = □

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

<u>Written methods (progressing to more than 4-digits)</u>

When understanding of the expanded method is secure, children will move on to the formal method of decomposition, which can be initially modelled with place value counters.



Progress to calculating with decimals, including those with different numbers of decimal places.

Year 6

Missing number/digit problems: □ and # each stand for a different number. # = 34. # + # = □ + □ + #. What is the value of □? What if # = 28? What if # = 21

10 000 000 = 9 000 100 + □

7 – 2 x 3 = □; (7 – 2) x 3 = □; (□ - 2) x 3 = 15

<u>Mental methods</u> should continue to develop, supported by a range of models and images, including the number line. The bar model should continue to be used to help with problem solving.

<u>Written methods</u>

As year 5, progressing to larger numbers, aiming for both conceptual understanding and procedural fluency with decomposition to be secured.

Subtract decimals to 3dp.

Continue calculating with decimals, including those with different numbers of decimal places up to 3dp.

Year 4

Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.
They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards: 124 – 47, count back 40 from 124, then 4 to 80, then 3 to 77
- Reordering: 75-28, (thinking of 28 as 25 + 3)
- Partitioning: counting on or back: 56-34, 56-30 then -4
- Partitioning: bridging through multiples of 10: 6070 – 4987, 4987 + 13 + 1000 + 70
- Partitioning: compensating – 138 - 69, 138 – 70 + 1
- Partitioning: bridging through 60 to calculate a time interval – What was the time 33 minutes before 2.15pm?
- Using known facts and place value to find related facts.

Year 5

Mental Strategies

Children should continue to count regularly, on and back, now including steps of powers of 10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged where appropriate.
Children should continue to partition numbers in different ways.
They should be encouraged to choose from a range of strategies:

- Counting forwards and backwards in tenths and hundredths: 1.7 + 0.55
- Reordering: 4.7 + 5.6 – 0.7, 4.7 – 0.7 + 5.6 = 4 + 5.6
- Partitioning: counting on or back - 540 + 280, 540 + 200 + 80
- Partitioning: bridging through multiples of 10:
- Partitioning: compensating: 5.7 + 3.9, 5.7 + 4.0 – 0.1
- Using known facts and place value to find related facts.
- Partitioning: bridging through 60 to calculate a time interval – What was the time 63 minutes before 2.45pm?

Year 6

Mental Strategies

Consolidate previous years.
Children should experiment with order of operations (BODMAS) (Brackets, orders, division or multiplication, addition or subtraction) investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45

Write different number sentences using the digits 2, 3, 5 and 8 before the equals sign, using:

- one operation
- two operations but no brackets
- two operations and brackets.

Can you write a number sentence using the digits 2, 3, 5 and 8 before the equals sign, which has the same answer as another number sentence using the digits 2, 3, 5 and 8 but which is a different sentence?

Year 4

Vocabulary
add, addition, sum, more, plus, increase, sum, total, altogether, double, near double, how many more to make..? how much more? ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, hundredths boundary, inverse, how many more/fewer? Equals sign, is the same as.

Generalisations
Investigate when re-ordering works as a strategy for subtraction. Eg. 20 – 3 – 10 = 20 – 10 – 3, but 3 – 20 – 10 would give a different answer.

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mastery skills

Demonstrate learning with concrete objects and pictorial representations.

Write three calculations where you would use mental calculation strategies and three where you would apply column method. Explain the decision you made for each calculation.

Year 5

Vocabulary
tens of thousands boundary,
Also see previous years

Generalisation
Use rounding to make approximate answers to calculations and determine levels of accuracy
Use knowledge of the order of operations to carry out calculations involving +, –, x and ÷

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mastery skills

Set out and solve these calculations using a column method.

3254 + ☐ = 7999

2431 = ☐ – 3456

6373 – ☐ = 3581

6719 = ☐ – 4562

True or False?

- 3999 – 2999 = 4000 – 3000
- 3999 – 2999 = 3000 – 2000
- 2741 – 1263 = 2742 – 1264
- 2741 + 1263 = 2742 + 1264
- 2741 – 1263 = 2731 – 1253
- 2741 – 1263 = 2742 – 1252

Explain your reasoning.

Using this number statement, 5222 – 3111 = 5223 – 3112 write three more pairs of equivalent calculations.

Year 6

Vocabulary
See previous years

Generalisations
Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.
Sometimes, always or never true? Subtracting numbers makes them smaller.
Multi step problems in contexts, choosing operation, method and explaining why.

Some Key Questions
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mastery skills

Two numbers have a difference of 2·38. What could the numbers be if:

- the two numbers add up to 6?
- one of the numbers is three times as big as the other number?

Two numbers have a difference of 2·3. To the nearest 10, they are both 10. What could the numbers be?

x and *y* represent whole numbers. Their sum is 1000.

Can the difference between *x* and *y* be:

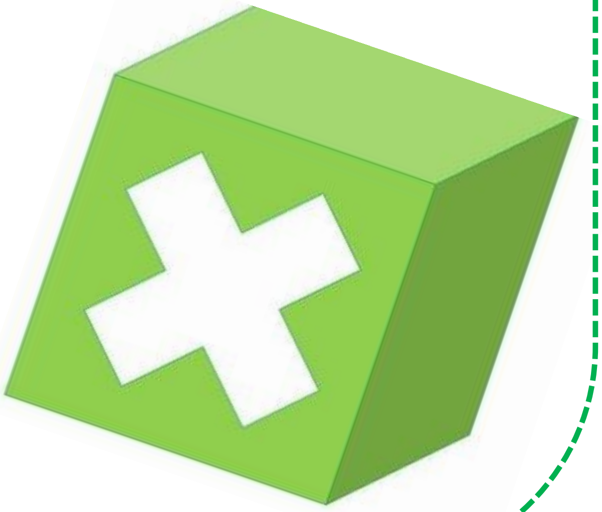
- 100?
- any whole number?
- greater than *x*?

	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5	YEAR 6
Hallam Fields Junior School Growing together, Learning together, Achieving together MULTIPLICATION AND DIVISION *Tables and Division facts and divide by 10Practical skills* Hallam Fields Junior School Growing together, Learning together, Achieving together	Count on and back in 1s, 2s, 5s & 10s.	Calculate mathematical statements for multiplication and division within times tables. Write multiplication and division sentences using x, ÷ and = signs. Solve x and ÷ problems using practical equipment including problems in context. Recall and use multiplication and division facts for 2,5 and 10 multiplication tables. Double all numbers up to 10 & halve all even numbers up to 20.	Recognise and explain patterns in division and multiplication. Demonstrate learning with concrete objects and own pictorial representations Solve x and ÷ problems including missing numbers and integer scaling problems. Recall and use 3, 4 & 8 multiplication and division facts. Multiply 1 & 2 digit numbers by 10 & 100. (using 0 as place value holder) Understand relationship between columns (x10, x100 for two). Multiply 2 digit numbers practically (as repeated addition). Awareness of inverse operations. Divide 2 digit numbers practically leading to simple division by grouping (no remainders) e.g. 25 divided by 5. Solve x and ÷ problems using concrete objects and pictorial representations Halve numbers related to x2 table. Double all numbers up to 20	Create own song / rhyme for remembering times tables. Use key vocabulary to explain method and reasoning to peers. Demonstrate learning with concrete objects and own pictorial representations Multiply & divide numbers up to 1000 by 2, 3, 4 or 5 & find remainders. Use place value, known and derived facts to x and ÷ mentally, multiplying together three numbers. Multiply two-digit and three-digit numbers by a one-digit number using formal written layout. Recall multiplication and division facts for multiplication tables up to 12 x 12. Estimate and use inverse to check Divide numbers up to 1000 by 10 or 100. X and divide any number by 10 and 100 up to HTU (using 0 as place value holder) Multiply & divide 2 digit numbers by 10 or 100. Recognise and use factor pairs and commutativity in mental calculations. Solve problems involving x and ÷. Halve numbers up to 50 (inc. numbers with odd number in tens) Double any number up to 50 Use place value to x mentally by 0 and 1; ÷ by 1.	Real life investigations applying scaling skills. Use key vocabulary to explain method and reasoning to peers. Demonstrate learning with concrete objects and own pictorial representations Recognise and use cube numbers and the notation for cubed. To solve x and ÷ problems, including scaling by simple fractions and problems involving simple ratios. Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy. Identify prime numbers up to 100. Recall prime numbers up to 19 Use vocabulary – prime numbers, prime factors and composite (non-prime) numbers. Divide numbers up to 4 digits by a one-digit number using formal written method of short division and interpret remainders appropriately for the context. Calculate halves & doubles of decimals. Identify multiples and factors, finding all factor pairs of a number, and common factors of two numbers. Recognise and use square numbers and the notation for squared. Use knowledge of the order of operations to carry out calculations involving the four operations. Multiply numbers up to 4 digits by a one- or two-digit number using formal written method, including long multiplication for two-digit numbers. X and ÷ whole numbers and those involving decimals by 10 , 100 and 1,000 Halve numbers up to 100 mentally. Double numbers to 100 mentally. Use tables to derive other number facts.	Use key vocabulary to explain method and reasoning to peers. Investigations, e.g. what would happen if? What is the relationship between…? Demonstrate learning with concrete objects and own pictorial representations Solve problems using ration, using multiplication and division facts. Solve problems requiring answers to be rounded to specified degrees of accuracy. Interpret remainders by rounding, as appropriate for the context. Use written division methods in cases where the answer has up to two decimal places. Multiply one-digit number with up to two decimal places by whole numbers. Calculate: U.t × U U.t ÷ U Use tables and place value to work with decimals (to 1dp). Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders. X numbers with 4 digits by 2 digits of a whole number Multiply 2 & 3 digit integers by 2-digit integer. Halve numbers up to 200 mentally. Double numbers up to 200 mentally. Identify prime numbers and square numbers. To identify common factors, common multiples and prime numbers. Interpret remainders as fractions. Calculate: TUxU TU÷U Use multiplication facts to find square numbers to 12x12. Multiply & divide decimals mentally by 10 or 100, & integers to 1000.

Hallam Fields Junior School

Growing together, Learning together, Achieving together

Multiplication

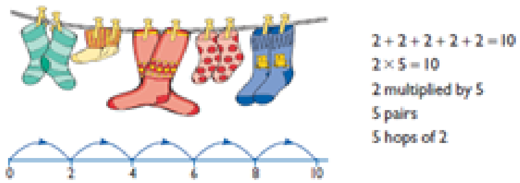


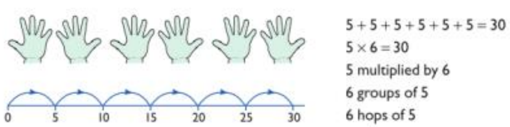
Year 1	Year 2	Year 3

Year 1

Understand multiplication is related to doubling and combing groups of the same size (repeated addition)

Washing line, and other practical resources for counting. Concrete objects. Numicon; bundles of straws, bead strings

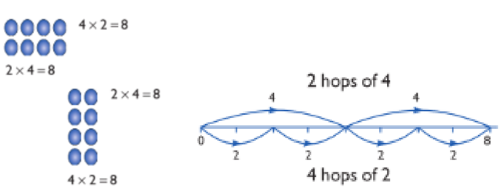




Problem solving with concrete objects (including money and measures

Use cuissenaire and bar method to develop the vocabulary relating to 'times' –
Pick up five, 4 times

Use arrays to understand multiplication can be done in any order (commutative)



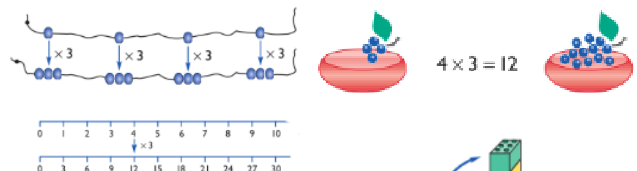
Year 2

Expressing multiplication as a number sentence using x
Using understanding of the inverse and practical resources to solve missing number problems.

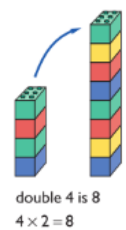
7 x 2 = □   □ = 2 x 7
7 x □ = 14   14 = □ x 7
□ x 2 = 14   14 = 2 x □
□ ⓧ = 14   14 = □ ⓧ

Develop understanding of multiplication using array and number lines (see Year 1). Include multiplications not in the 2, 5 or 10 times tables.

Begin to develop understanding of multiplication as scaling (3 times bigger/taller)

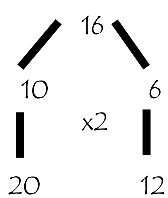


Doubling numbers up to 10 + 10
Link with understanding scaling
Using known doubles to work out double 2d numbers
(double 15 = double 10 + double 5)



Towards written methods

Use jottings to develop an understanding of doubling two digit numbers.



Year 3

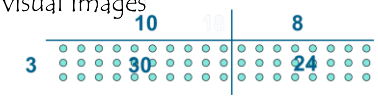
Missing number problems
Continue with a range of equations as in Year 2 but with appropriate numbers.

Mental methods

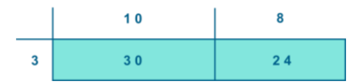
Doubling 2 digit numbers up to 20 using partitioning
Demonstrating multiplication on a number line – jumping in larger groups of amounts
13 x 4 = 10 groups 4 = 3 groups of 4

Written methods (progressing to 2d x 1d)

Multiply 2 digit numbers practically as repeated addition.
Developing written methods using understanding of visual images



Develop on to the grid method



Give children opportunities for children to explore this and deepen understanding using Dienes apparatus and place value counters

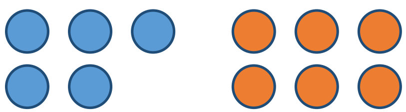
Mental Strategies

Children should continue to count regularly, on and back, now including multiples of 4, 8, 50, and 100, and steps of 1/10.
The number line should continue to be used as an important image to support thinking, and the use of informal jottings and drawings to solve problems should be encouraged.
Children should practise times table facts
3 x 1 =   3 x 2 =   3 x 3 =
Multiply 1digit and 2digit numbers by 10/100 – making numbers 10/100 times bigger and using 0 as a place holder.

Year 1

Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.
Children should memorise and reason with numbers in 2, 5 and 10 times tables
They should see ways to represent odd and even numbers. This will help them to understand the pattern in numbers.



Children should begin to understand multiplication as scaling in terms of double and half. (e.g. that tower of cubes is double the height of the other tower)

Vocabulary

Ones, groups, lots of, doubling
repeated addition
groups of, lots of, times, columns, rows
longer, bigger, higher etc
times as (big, long, wide …etc)

Generalisations

Understand 6 counters can be arranged as 3+3 or 2+2+2
Understand that when counting in twos, the numbers are always even.

Some Key Questions

Why is an even number an even number?
What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Year 2

Mental Strategies

Children should count regularly, on and back, in steps of 2, 3, 5 and 10.
Number lines should continue to be an important image to support thinking, for example
Children should practise times table facts
2 x 1 =
2 x 2 =
2 x 3 =
Use a clock face to support understanding of counting in 5s.
Use money to support counting in 2s, 5s, 10s, 20s, 50s

Vocabulary

multiple, multiplication array, multiplication tables / facts
groups of, lots of, times, columns, rows

Generalisation

Commutative law shown on array (video)
Repeated addition can be shown mentally on a number line
Inverse relationship between multiplication and division. Use an array to explore how numbers can be organised into groups.

Some Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Year 3

Vocabulary

partition
grid method
inverse
Place holder

Generalisations

Connecting x2, x4 and x8 through multiplication facts
Comparing times tables with the same times tables which is ten times bigger. If 4 x 3 = 12, then we know 4 x 30 = 120. Use place value counters to demonstrate this.
When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?)
Solve multiplication problems using concrete objects and pictorial representations.
Awareness of inverse operations.

Some Key Questions

What do you notice?
What's the same? What's different?
Can you convince me?
How do you know?

Mastery skills

What is the relationship between these calculations?

2 × 3	4 × 3
2 × 30	4 × 30
20 × 3	40 × 3
20 × 3 × 10	40 × 3 × 10

☐☐ × ☐ = ?

Putting the digits 1, 2 and 3 in the empty boxes, how many different calculations can you make?

Which one gives the largest answer?
Which one gives the smallest answer?

Year 4	Year 5	Year 6

Year 4

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits
□2 x 5 = 160

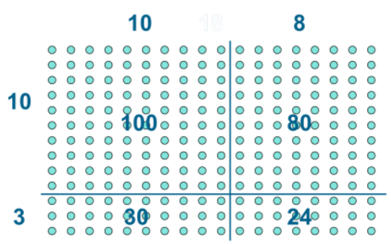
<u>Mental methods</u>
Counting in multiples of 6, 7, 9, 25 and 1000, and steps of 1/100.

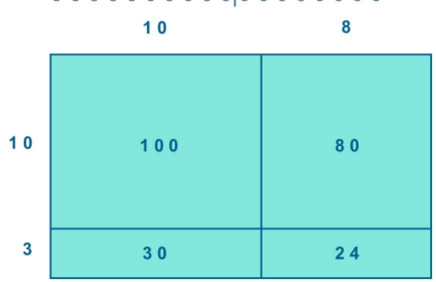
Solving practical problems where children need to scale up. Relate to known number facts. (e.g. how tall would a 25cm sunflower be if it grew 6 times taller?)

Double numbers up to 50.

Recognise and use factor pairs and commutativity.

<u>Written methods (progressing to 3d x 2d)</u>
Children to embed and deepen their understanding of the grid method to multiply up 2d x 2d. Ensure this is still linked back to their understanding of arrays and place value counters.





Year 5

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

<u>Mental methods</u>
X by 10, 100, 1000 – making numbers 10/100/1000 times bigger and using 0 as a place holder.

Use practical resources and jottings to explore equivalent statements (e.g. 4 x 35 = 2 x 2 x 35)

Recall of prime numbers up to 19 and identify prime numbers up to 100 (with reasoning)

Solving practical problems where children need to scale up. Relate to known number facts.

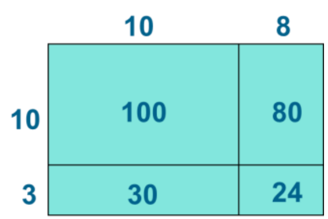
Identify factor pairs for numbers

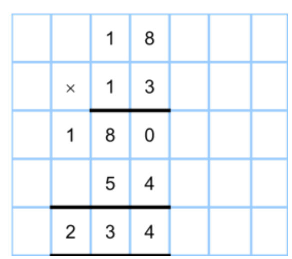
Halve numbers up to 100

<u>Written methods (progressing to 4d x 2d)</u>

Long multiplication using place value counters

Children to explore how the grid method supports an understanding of long multiplication (for 2d x 2d)





Year 6

Continue with a range of equations as in Year 2 but with appropriate numbers. Also include equations with missing digits

<u>Mental methods</u>
Double numbers to 200
Identifying common factors and common multiples of given numbers
Solving practical problems where children need to scale up. Relate to known number facts.
Multiply decimals by 10/100 and integers to 1000

<u>Written methods (progressing to 4d x 2d and U.th x U</u>
Continue to refine and deepen understanding of written methods including fluency for using long multiplication
Develop written method from grid:

X	1000	300	40	2
10	10000	3000	400	20
8	8000	2400	320	16

```
  1 3 4 2
x     1 8
1 3 4 2 0
1 0 7 3 6
2 4 1 5 6
1
```

Develop standard long multiplication method:

```
  612          423
X  24       X  211
 2448          423
12240         4230
14688        84600
             89253
               1
```

Year 4	Year 5	Year 6
Mental Strategies Children should continue to count regularly, on and back, now including multiples of 6, 7, 9, 25 and 1000, and steps of 1/100. Become fluent and confident to recall all tables to x 12 Use the context of a week and a calendar to support the 7 times table (e.g. how many days in 5 weeks?) Use of finger strategy for 9 times table. Multiply 3 numbers together Multiply 2digit numbers by 10 and 100 The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication **Vocabulary** Factor **Generalisations** Children given the opportunity to use place value to investigate numbers multiplied by 1 and 0. When they know multiplication facts up to x12, do they know what x13 is? (i.e. can they use 4x12 to work out 4x13 and 4x14 and beyond?) Estimate and use inverse operation to check. Solve problems involving x and + **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?	**Mental Strategies** Children should continue to count regularly, on and back, now including steps of powers of 10. Multiply by 10, 100, 1000, including decimals. Use knowledge of order of operations to carry out calculations involving 4 operations. The number line should continue to be used as an important image to support thinking, and the use of informal jottings should be encouraged. They should be encouraged to choose from a range of strategies to solve problems mentally: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 13 times tables or the 24 times table) **Vocabulary** cube numbers prime numbers square numbers common factors prime number, prime factors, composite numbers **Generalisation** Relating arrays to an understanding of square numbers and making cubes to show cube numbers. Understanding that the use of scaling by multiples of 10 can be used to convert between units of measure (e.g. metres to kilometres means to times by 1000) Use rounding to check and determine levels of accuracy **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know? How do you know this is a prime number?	**Mental Strategies** Consolidate previous years. Children should experiment with order of operations (BODMAS), investigating the effect of positioning the brackets in different places, e.g. 20 – 5 x 3 = 5; (20 – 5) x 3 = 45 They should be encouraged to choose from a range of strategies to solve problems mentally: - Partitioning using x10, x20 etc - Doubling to solve x2, x4, x8 - Recall of times tables - Use of commutativity of multiplication If children know the times table facts to 12 x 12. Can they use this to recite other times tables (e.g. the 24 times tables or the 36 times table) Estimating as a strategy for checking the accuracy of calculations. **Vocabulary** See previous years common factor **Generalisations** Order of operations – Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering. Understanding the use of multiplication to support conversions between units of measurement. **Some Key Questions** What do you notice? What's the same? What's different? Can you convince me? How do you know?

Year 4

Mastery skills

True or false?

$7 \times 6 = 7 \times 3 \times 2$

$7 \times 6 = 7 \times 3 + 3$

Explain your reasoning.

Three children calculated 7×6 in different ways.
Identify each strategy and complete the calculations.

Annie	Bertie	Cara used the commutative law
$7 \times 6 = 7 \times 5 + \square$ $= \square$	$7 \times 6 = 7 \times 7 - \square$ $= \square$	$7 \times 6 = \square \times \square$ $= \square$

Now find the answer to 6×9 in three different ways.

Tom ate 9 grapes at the picnic. Sam ate 3 times as many grapes as Tom.
How many grapes did they eat altogether?

The bar model is a useful scaffold to develop fluency in this type of question.

Place one of these symbols in the circle to make the number sentence correct: >, < or =.

Explain your reasoning.

8×50 ◯ 50×8

8×50 ◯ 80×5

300×3 ◯ 5×200

What do you notice about the following calculations? Can you use one calculation to work out the answer to other calculations?

$2 \times 3 =$	$6 \times 7 =$	$9 \times 8 =$
$2 \times 30 =$	$6 \times 70 =$	$9 \times 80 =$
$2 \times 300 =$	$6 \times 700 =$	$9 \times 800 =$
$20 \times 3 =$	$60 \times 7 =$	$90 \times 8 =$
$200 \times 3 =$	$600 \times 7 =$	$900 \times 8 =$

Year 5

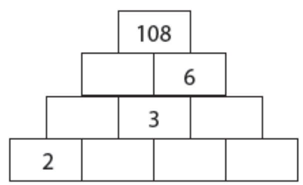
Mastery skills

8	is a multiple of	4	and a factor of	16
6	is a multiple of	3	and a factor of	$\square$
$\square$	is a multiple of	5	and a factor of	$\square$
$\square$	is a multiple of	$\square$	and a factor of	$\square$

Captain Conjecture says, 'Factors come in pairs so all numbers have an even number of factors.'

Do you agree?
Explain your reasoning.

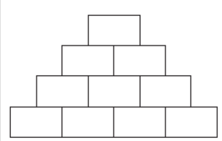
Fill in the missing numbers in this multiplication pyramid.



Put the numbers 1, 2, 3 and 4 in the bottom row of this multiplication pyramid in any order you like.

What different numbers can you get on the top of the number pyramid? How can you make the largest number?

Explain your reasoning.



Year 6

Mastery skills

Fill in the missing numbers to make these number sentences true.

$\square \times \square = 864$

$\square \times \square \times \square = 864$

It is correct that $273 \times 32 = 8736$. Use this fact to work out:

- $27{\cdot}3 \times 3{\cdot}2$
- $2{\cdot}73 \times 32000$
- $873{\cdot}6 \div 0{\cdot}32$
- $87{\cdot}36 \div 27{\cdot}3$
- $8736 \div 16$
- $4368 \div 1{\cdot}6$

Which calculation is the odd one out?

- $753 \times 1{\cdot}8$
- $(75{\cdot}3 \times 3) \times 6$
- $753 + 753 \div 5 \times 4$
- $7{\cdot}53 \times 1800$
- $753 \times 2 - 753 \times 0{\cdot}2$
- $750 \times 1{\cdot}8 + 3 \times 1{\cdot}8$

Explain your reasoning.

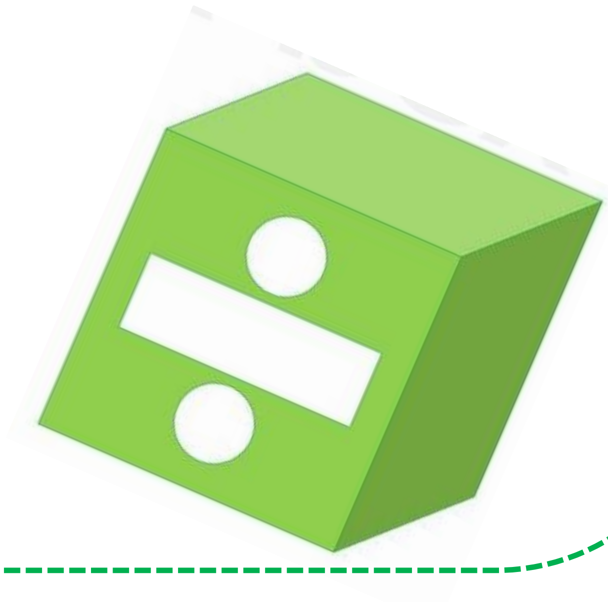
Miriam and Alan each buy 12 tins of tomatoes.
Miriam buys 3 packs each containing 4 tins. A pack of 4 costs £1·40.
Alan buys 2 packs each containing 6 cans. A pack of 6 costs £1·90.

Who gets the most change from a £5 note?

Hallam Fields Junior School

Growing together, Learning together, Achieving together

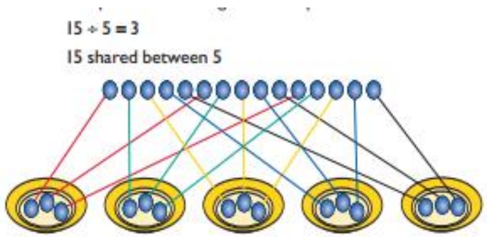
Division



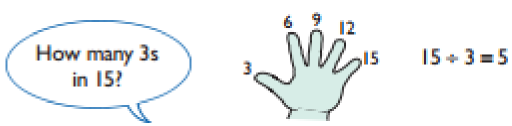
Year 1

Children must have secure counting skills- being able to confidently count in 2s, 5s and 10s.
Children should be given opportunities to reason about what they notice in number patterns.

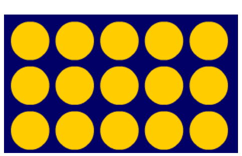
<u>Group AND share small quantities- understanding the difference between the two concepts.</u>
<u>Sharing</u>
Develops importance of one-to-one correspondence.

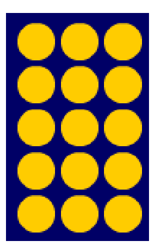


Children should be taught to share using concrete apparatus.
<u>Grouping</u>
Children should apply their counting skills to develop some understanding of grouping.



Use of arrays as a pictorial representation for division.
15 ÷ 3 = 5 There are 5 groups of 3.
15 ÷ 5 = 3 There are 3 groups of 5.





Children should be able to find ½ and ¼ and simple fractions of objects, numbers and quantities.

Year 2

<u>÷ = signs and missing numbers</u>

6 ÷ 2 = □	□ = 6 ÷ 2
6 ÷ □ = 3	3 = 6 ÷ □
□ ÷ 2 = 3	3 = □ ÷ 2
□ ÷ ∇ = 3	3 = □ ÷ ∇

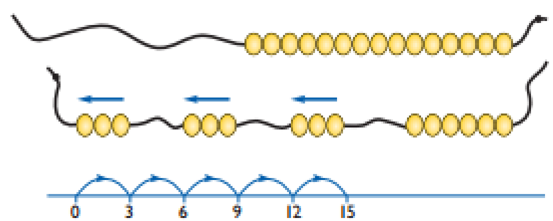
Know and understand sharing and grouping- introducing children to the ÷ sign.

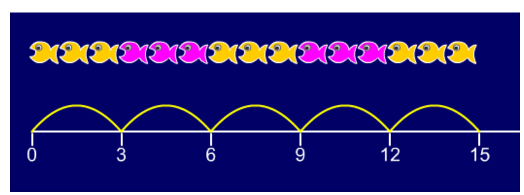
Children should continue to use grouping and sharing for division using practical apparatus, arrays and pictorial representations.

<u>Grouping using a numberline</u>

Group from zero in jumps of the divisor to find our 'how many groups of 3 are there in 15?'.

15 ÷ 3 = 5

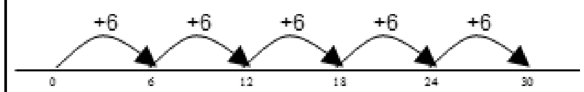




Continue work on arrays. Support children to understand how multiplication and division are inverse. Look at an array – what do you see?

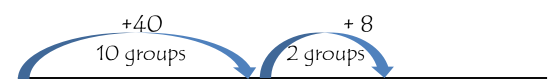
Year 3

<u>÷ = signs and missing numbers</u>
Continue using a range of equations as in year 2 but with appropriate numbers.
Divide 2digit numbers practically
Develop to simple division by <u>grouping</u> without remainders.
How many 6's are in 30?
30 ÷ 6 can be modelled as:

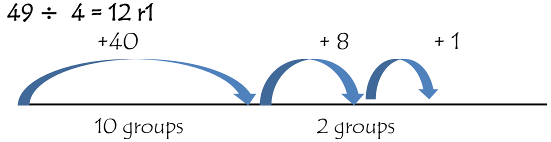


<u>Becoming more efficient using a number line</u>

Children need to be able to partition the dividend in different ways.
48 ÷ 4 = 12



Develop to division with <u>remainders</u>
49 ÷ 4 = 12 r1



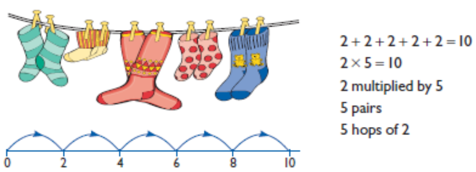
Sharing – 49 shared between 4. How many left over?
Grouping – How many 4s make 49. How many are left over?

Place value counters can be used to support children apply their knowledge of grouping.
For example:
60 ÷ 10 = How many groups of 10 in 60?
600 ÷ 100 = How many groups of 100 in 600?

Year 1

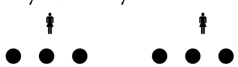
Mental Strategies

Children should experience regular counting on and back from different numbers in 1s and in multiples of 2, 5 and 10.
They should begin to recognise the number of groups counted to support understanding of relationship between multiplication and division.



Children should begin to understand division as both sharing and grouping.

Sharing – 6 sweets are shared between 2 people.
How many do they have each?



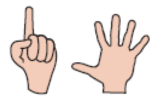
Grouping–
How many 2's are in 6?

They should use objects to group and share amounts to develop understanding of division in a practical sense.
E.g. using Numicon to find out how many 5's are in 30? How many pairs of gloves if you have 12 gloves?
Children should begin to explore finding simple fractions of objects, numbers and quantities.
E.g. *16 children went to the park at the weekend. Half that number went swimming. How many children went swimming?*

Year 2

Mental Strategies

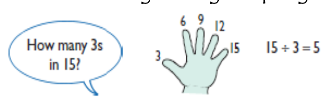
Children should count regularly, on and back, in steps of 2, 3, 5 and 10.
Children who are able to count in twos, threes, fives and tens can use this knowledge to work out other facts such as 2 × 6, 5 × 4, 10 × 9. Show the children how to hold out their fingers and count, touching each finger in turn. So for 2 × 6 (six twos), hold up 6 fingers:



Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

Touching the fingers in turn is a means of keeping track of how far the children have gone in creating a sequence of numbers. The physical action can later be visualised without any actual movement.

This can then be used to support finding out 'How many 3's are in 18?' and children count along fingers in 3's therefore making link between multiplication and division.
Children should continue to develop understanding of division as sharing **and** grouping.



15 pencils shared between 3 pots, how many in each pot?
Children should be given opportunities to find a half, a quarter and a third of shapes, objects, numbers and quantities. Finding a fraction of a number of objects to be related to sharing.
They will explore visually and understand how some fractions are equivalent – e.g. two quarters is the same as one half.

Year 3

Mental Strategies

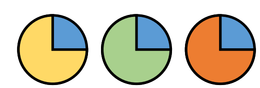
Children should count regularly, on and back, in steps of 3, 4 and 8.

Halve numbers related to 2 x tables

Children will make use multiplication and division facts they know to make links with other facts.
3 x 2 = 6, 6 ÷ 3 = 2, 2 = 6 ÷ 3
30 x 2 = 60, 60 ÷ 3 = 20, 2 = 60 ÷ 30

They should be given opportunities to solve grouping and sharing problems practically (including where there is a remainder but the answer needs to given as a whole number)
e.g. Pencils are sold in packs of 10. How many packs will I need to buy for 24 children?

Children should be given the opportunity to further develop understanding of division (sharing) to be used to find a fraction of a quantity or measure.
Use children's intuition to support understanding of fractions as an answer to a sharing problem.
3 apples shared between 4 people = $^3/_4$



Year 1

<u>Vocabulary</u>

share, share equally, one each, two each…, group, groups of, lots of, array

<u>Generalisations</u>

- True or false? I can only halve even numbers.
- Grouping and sharing are different types of problems. Some problems need solving by grouping and some by sharing. Encourage children to practically work out which they are doing.

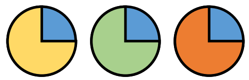
<u>Some Key Questions</u>

How many groups of…?
How many in each group?
Share… equally into…
What can do you notice?

Year 2

<u>Use children's intuition to support understanding of fractions as an answer to a sharing problem.</u>

3 apples shared between 4 people = $\frac{3}{4}$



<u>Vocabulary</u>

group in pairs, 3s … 10s etc
equal groups of
divide, ÷, divided by, divided into, remainder

<u>Generalisations</u>

Noticing how counting in multiples if 2, 5 and 10 relates to the number of groups you have counted (introducing times tables)
An understanding of the more you share between, the less each person will get (e.g. would you prefer to share these grapes between 2 people or 3 people? Why?)
Secure understanding of grouping means you count the number of groups you have made. Whereas sharing means you count the number of objects in each group.

<u>Some Key Questions</u>

How many 10s can you subtract from 60?
I think of a number and double it. My answer is 8. What was my number?
If 12 x 2 = 24, what is 24 ÷ 2?
Questions in the context of money and measures (e.g. how many 10p coins do I need to have 60p? How many 100ml cups will I need to reach 600ml?)

Year 3

<u>Vocabulary</u>

See Y1 and Y2
inverse

<u>Generalisations</u>

Inverses and related facts – develop fluency in finding related multiplication and division facts.
Develop the knowledge that the inverse relationship can be used as a checking method.

Solve division problems using concrete objects and pictorial representations.

<u>Some Key Questions</u>

Questions in the context of money and measures that involve remainders (e.g. How many lengths of 10cm can I cut from 81cm of string? You have £54. How many £10 teddies can you buy?)
What is the missing number? 17 = 5 x 3 + __
__ = 2 x 8 + 1

<u>Mastery skills</u>

Roger has 96 patio slabs.
Using all of the slabs find three different ways that he can arrange the slabs to form a rectangular patio.

Write a story for 18 ÷ 3.

Year 4

÷ = signs and missing numbers
Continue using a range of equations as in year 3 but with appropriate numbers.

Sharing, Grouping and using a number line
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line until they have a secure understanding. Children should progress in their use of written division calculations:

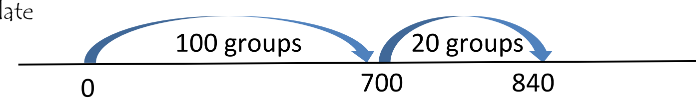
- Using tables facts with which they are fluent
- Experiencing a logical progression in the numbers they use, for example:

1. Dividend just over 10x the divisor, e.g. 84 ÷ 7
2. Dividend just over 10x the divisor when the divisor is a teen number, e.g. 173 ÷ 15 (learning sensible strategies for calculations such as 102 ÷ 17)
3. Dividend over 100x the divisor, e.g. 840 ÷ 7
4. Dividend over 20x the divisor, e.g. 168 ÷ 7

All of the above stages should include calculations with remainders as well as without.
Remainders should be interpreted according to the context. (i.e. rounded up or down to relate to the answer to the problem)

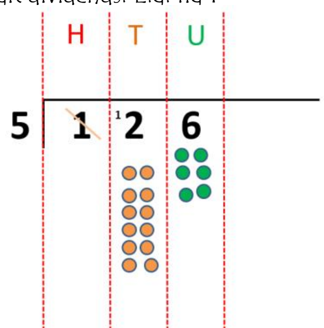
e.g. 840 ÷ 7 = 120

Jottings
7 x 100 = 700
7 x 10 = 70
7 x 20 = 140



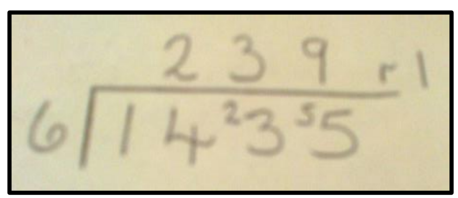
Formal Written Methods
Formal short division should only be introduced once children have a good understanding of division, its links with multiplication and the idea of 'chunking up' to find a target number (see use of number lines above)

Short division to be modelled for understanding using place value counters as shown below. Calculations with 2 and 3-digit dividends. E.g. fig 1



Year 5

Formal Written Methods (progressing to 4d x 1d)
Continued as shown in Year 4, leading to the efficient use of a formal method. The language of grouping to be used
E.g. 1435 ÷ 6



Children begin to practically develop their understanding of how express the remainder as a decimal or a fraction. Ensure practical understanding allows children to work through this (e.g. what could I do with this remaining 1? How could I share this between 6 as well?)

Year 6

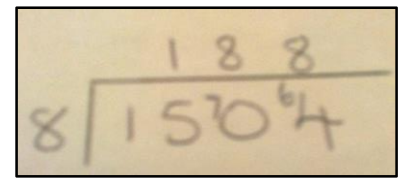
÷ = signs and missing numbers
Continue using a range of equations but with appropriate numbers

Sharing and Grouping and using a number line
Children will continue to explore division as sharing and grouping, and to represent calculations on a number line as appropriate.

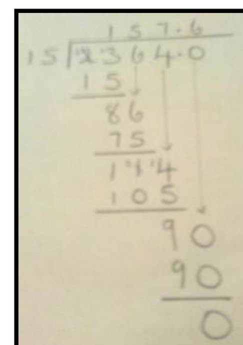
Quotients should be expressed as decimals and fractions

Formal Written Methods – long and short division (progressing to 4d x 2d)

E.g. 1504 ÷ 8



E.g. 2364 ÷ 15



Year 4

Mental Strategies

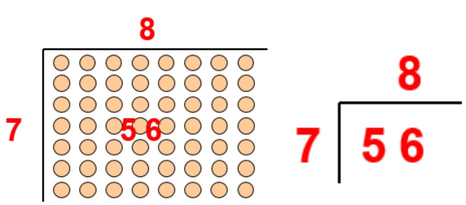
Children should experience regular counting on and back from different numbers in multiples of 6, 7, 9, 25 and 1000.
Children should learn the multiplication facts to 12 x 12.
Halve numbers to 50 including numbers with odd numbers in the tens.
Divide any number by 10/100

Vocabulary

see years 1-3
divide, divided by, divisible by, divided into
share between, groups of
factor, factor pair, multiple
times as (big, long, wide …etc)
equals, remainder, quotient, divisor
inverse

Towards a formal written method

Alongside pictorial representations and the use of models and images, children should progress onto short division using a bus stop method.



Place value counters can be used to support children apply their knowledge of grouping. Reference should be made to the value of each digit in the dividend.

Year 5

Mental Strategies

Children should count regularly using a range of multiples, and powers of 10, 100 and 1000, building fluency.
Children should practice and apply the multiplication facts to 12 x 12.
Halve numbers to 100
Divide whole numbers by 10 / 100 / 1000

Vocabulary

see year 4
common factors
prime number, prime factors
composite numbers
short division
square number
cube number
inverse
power of

Year 6

Mental Strategies

Children should count regularly, building on previous work in previous years.
Children should practice and apply the multiplication facts to 12 x 12. and derive division facts from these.
Halve numbers to 200
Divide decimals by 10 / 100 and integers to 1000

Vocabulary

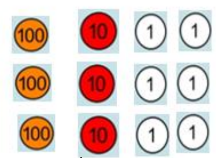
see years 4 and 5

Year 4

Each digit as a multiple of the divisor

'How many groups of 3 are there in the hundreds column?'
'How many groups of 3 are there in the tens column?'
'How many groups of 3 are there in the units/ones column?'

  1 1 2
3 | 3 3 6



When children have conceptual understanding and fluency using the bus stop method without remainders, they can then progress onto 'carrying' their remainder across to the next digit.

Generalisations

Inverses and deriving facts. 'Know one, get lots free!' e.g.: 2 x 3 = 6, so 3 x 2 = 6, 6 ÷ 2 = 3, 60 ÷ 20 = 3, 600 ÷ 3 = 200 etc.
Sometimes, always, never true questions about multiples and divisibility. (When looking at the examples on this page, remember that they may not be 'always true'!) E.g.:

- Multiples of 5 end in 0 or 5.
- The digital root of a multiple of 3 will be 3, 6 or 9.
- The sum of 4 even numbers is divisible by 4.

Mastery skills

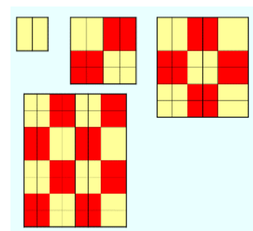
True or false? Dividing by 10 is the same as dividing by 2 and then dividing by 5. Can you find any more rules like this?
Is it sometimes, always or never true that □ ÷ Δ = Δ ÷ □?

Year 5

Generalisations

The = sign means equality. Take it in turn to change one side of this equation, using multiplication and division, e.g.
Start: **24 = 24**
Player 1: **4 x 6 = 24**
Player 2: **4 x 6 = 12 x 2**
Player 1: **48 ÷ 2 = 12 x 2**
Sometimes, always, never true questions about multiples and divisibility. E.g.:

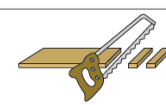
- If the last two digits of a number are divisible by 4, the number will be divisible by 4.
- If the digital root of a number is 9, the number will be divisible by 9.
- When you square an even number the result will be divisible by 4 (one example of 'proof' shown left)



Mastery skills

A 50 cm length of wood is cut into 4 cm pieces.

How many 4 cm pieces are cut and how much wood is left over?



Fill in the blanks to represent the problem as division:
□ ÷ □ = □ remainder □

Fill in the blanks to represent the problem as multiplication:
□ × □ + □ = 50

A 1 m piece of ribbon is cut into equal pieces and a piece measuring 4 cm remains.

What might the lengths of the equal parts be?

In how many different ways can the ribbon be cut into equal pieces?

Year 6

Generalisations

Order of operations: brackets first, then multiplication and division (left to right) before addition and subtraction (left to right). Children could learn an acrostic such as BODMAS, or could be encouraged to design their own ways of remembering.

Sometimes, always, never true questions about multiples and divisibility. E.g.: If a number is divisible by 3 and 4, it will also be divisible by 12. (also see year 4 and 5, and the hyperlink from the Y5 column)

Using what you know about rules of divisibility, do you think 7919 is a prime number? Explain your answer.

Mastery skills

In each pair of calculations, which one would you prefer to work out?

- (a) 35 × 0·3 + 35 × 0·7 **or** (b) 3·5 × 0·3 + 35 × 7
- (c) 6·4 × 1·27 – 64 × 0·1 **or** (d) 6·4 × 1·27 – 64 × 0·027
- (e) 52·4 ÷ 0·7 + 524 ÷ 7 **or** (f) 52·4 ÷ 0·7 – 524 ÷ 7
- (g) 31·2 ÷ 3 – 2·4 ÷ 6 **or** (h) 31·2 ÷ 3 – 1·2 ÷ 0·3

Explain your choices.

It is correct that 273 × 32 = 8736. Use this fact to work out:

- 27·3 × 3·2
- 2·73 × 32 000
- 873·6 ÷ 0·32
- 87·36 ÷ 27·3
- 8736 ÷ 16
- 4368 ÷ 1·6

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Mathematical Questions

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How could you sort these.......? How many ways can you find to .... ? What happens when we .... ? What can be made from....? How many different .... can be found? What do you notice? What is the same? Can you convince me? How do you know? Do you need to estimate first? Can you do this calculation mentally? Do you need to use a written method? What is different? Can you group these .... in some way? Can you see a pattern? Have you thought of another way this could be done?	How can this pattern help you find an answer? What do think comes next? Why? Is there a way to record what you've found that might help us see more patterns? What would happen if....? What have you discovered? How did you find that out? Why do you think that? What made you decide to do it that way? Who has the same answer/ pattern/ grouping as this? Who has a different solution? Are everybody's results the same? Why/why not? Have we found all the possibilities? How do we know? Do you think we have found the best solution?

Most teachers waste their time finding questions which are intended to discover what a pupil does not know. The true art of questioning has for it's purpose to discover what pupils know or are capable of knowing.

–Albert Einstein –